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ABSTRACT

The problem of selecting which sites to use in locating library and information centers in order to best achieve system objectives is modeled as a mixed zero-one linear programming problem and examined in detail. The model incorporates an objective function based on the costs of locating centers and of transmitting information to the users. The model also incorporates constraints to account for the scarcity of service available at the various sites (capacity) and for the need to at least satisfy the minimal demands of the users. The model is meant to be used to aid the library system's management in making better location decisions, since the objective function can be shown to reflect the evaluation of the library as an information retrieval system and as a service organization. Moreover, the model is seen to be related to the problems of locating plants and warehouses. A reformulation incorporating the properties of both interlibrary communication networks and the transmission of information services to clients is presented and discussed. The dual problem is derived and analyzed to yield several economic implications of the locational model for librarians. (Author)

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A Mathematical Model of the Information Center Location Problem and Its Implications for Decision Makers

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ABSTRACT

The problem of selecting which sites to use in locating library and information centers in order to best achieve system objectives is modeled as a mixed zero-one linear programming problem and examined in detail. The model incorporates an objective function based on the costs of locating the centers and of transmitting information service to the users. The model also incorporates constraints to account for the scarcity of service available at the various sites (capacity) and for the need to at least satisfy the minimal demands of the users. The model is meant to be used to aid the library system's management in making better location decisions, since the objective function can be shown to reflect the evaluation of the library as an information retrieval system and as a service organization. Moreover, the model is seen to be related to the problems of locating plants and warehouses.

A reformulation incorporating the properties of both interlibrary communication networks and the transmission of information services to clients is presented and discussed. The dual problem is derived and analyzed to yield several economic implications of the locational model for librarians.

Introduction

Consider the information center location problem in a library or information system. We shall investigate, in this paper, the problem of choosing from a list of possible sites those locations which seem most worthwhile in terms of the overall goals of our system. Much of the library literature [17, 22, 31, 39, 48, 61, 62] deals only with establishing rules-of-thumb based on prior experience and expert opinion. An overall model of the location decision process based on similar models for location of other types of facilities will be constructed here, and a discussion of the model's implications will follow.

While the model may be applicable to several types of information systems, we shall direct our attention and focus toward the location of information sources, even though much of the literature [6, 17, 22, 61, 62] deals with the location of main and branch libraries in public library systems.

The scope of the model requires a complete list of potential sites, from which we shall select a subset where the be located. We shall consider type, size, and capacity of the physical structures to provide various forms of service at any site, as well as client types, their locations, and their demands for service.

The importance of the problem has been reflected in the literature [10, 17, 22, 25, 39, 48, 52] ; especially by Wheeler [61, 62] who pointed out that libraries (public) may be second only to schools in importance to community improvement and that location is a major factor in the use of these libraries. Coughlin, Taieb, and Stevens [10] agree, especially for adult users. Moreover, a location decision has far reaching implications for the long range, and a poor decision may be very costly if not impossible to correct [61] . In general however, many of the studies [10, 17, 22, 61, 62] reflect only the placing of a main library or a few branches based on a small set of alternative sites in a particular situation, with no general,

overall decision model of the problem in its entirety.

The location problem for library systems has been compared to that of retail stores, especially bookstores [61] which opens up a whole body of literature [2,30,48] on retail facilities location problems. Moreover, economic geographers have also considered the problem of general economic facility location, especially from a descriptive point of view, so that another body of literature is available [28,29,38,47,52]. There is also the library network literature [8, 12, 13, 14, 27, 32, 42, 43, 44, 46, 51, 60].

Information System Decision Making

As Wagner [58] points out, decision-making is the major purpose behind much operations research or systems analysis work. In order to build an overall decision model relating to library location, we must define our variables in algebraic terms. Let L be the complete set of all possible sites under consideration, with m being the number of sites in the set and the subscript i (i = 1, 2,... m) representing the specific site i. Let N be the set of all locations where users exist, with n being the number of user locations and the subscript j (j = 1, 2,... n) representing the specific user location j. A user location is a specific point where users exist, or a center point in the middle of a cluster of users; for example, the center of a city block where many users live. Moreover, while this procedure implies that the sites and user locations have been enumerated, no rank order is to be inferred. Finally, only those sites in L will be considered.

We can now define our decision variables as those variables which represent our decision (z) of whether or not to place a library at a specific site and our decision (x) of how much service a library at that site will give to users at a specific location.

The term "library" will be used throughout the rest of this paper to represent the facility to be considered for placement at a given site, whether it is a library in the traditional sense or an information center with modern telecommunication links. Let z_i = a binary-valued decision variable, set equal to unity if a library is placed at site i, and equal to zero if not (it is with the optimal values for the z_i variables that will be most concerned), and x_{ij} = a con-

tinuous and nonnegative variable, representing the amount of service a library at site i can give to users at location j. Service must be measured in units that reflect the total level of service given in all its various forms (reference, material circulation, crisis information, etc.). This idea of a general unit of service is indeed an artificial construct, but it is quite useful, as we shall see; and it may be possible to define [37]. Nance [42], for example, uses the message as a unit of service in his analytical model of a library network. A given set of values for the z_i and x_{ij} variables represents one alternative location policy in terms of where to locate libraries in a library system. A different set of values yields a different selection policy. That is, the set of decision variables, denoted as (x,z), tells us which sites are or are not to be used and to what extent. Subsequently, in this paper we will say "Let $z_i = 1$", or "Let z_i take the value of 1", instead of "Place a library at site i." Also, we shall now say "Let $x_{ij} = x$", where x is some specific value for x_{ij}, instead of "Let a library at site i transmit x units of service to users at location j."

We now need to define criteria to help us decide which sites to use, i.e., which z_i's will be set equal to one, and to what extent will the sites be used, i.e., what the values for the x_{ij}'s should be. Moreover, we need to define which factors restrain us from locating at just any site, factors such as user demand for service which must be met and limited site capacity for the provision of service.

The Objective Function

In order to determine which location policy is best, (i.e., which values for the members of (x,z) are best), we need to define the goals and objectives of our system [1c]. If we can then establish criteria in order to measure how effective a specific policy is, we can evaluate that policy in terms of how well it helps us to achieve our goals. Moreover, we can compare

the alternative location policies on the basis of such an evaluation and choose the best or optimal policy.

To do this we must first define the term "site cost" as the measured cost, in terms of using up scarce resources such as limited capital, of placing a library at site i, and the term "transport cost" as the measured cost of delivering one unit of service from a library at site i to users at location j. Let f_i = the expected site cost to place a library at site i, and c_{ij} = the expected transport cost to deliver one unit of service from a library at site i to users at location j. We note that the expression $f_i z_i$ = the site cost to place a library at site i actually expended, and that the expression $c_{ij}x_{ij}$ = the total transport cost to provide service from a library at site i to users at location j. Of course, both c_{ij} and f_i are nonnegative, for all values of i and j. Summing these costs together for all possible sites and user locations yields a total cost which we wish to make as small as possible. Thus we wish to

(1) Minimize $\sum_{j=1}^{n}\sum_{i=1}^{m} c_{ij}x_{ij} + \sum_{i=1}^{m} f_i z_i$.

Note the use of the sum sign as defined in the example below:

$$\sum_{j=1}^{n} a_j = a_1 + a_2 + \ldots + a_j + \ldots + a_{n-1} + a_n.$$

We make this objective function as small as possible by acquiring good sites for our libraries (letting $z_i = 1$ for those sites deemed inexpensive and by choosing proper values for the x_{ij} variables).

The library has been viewed as an information storage and retrieval system [7] and as a service organization [3]. The model, as we shall see, reflects both views, in terms of evaluation of retrieval systems and in terms of the inputs of the service organization participants.

The costs of location are to be minimized, which is directly in line with the idea of an efficient retrieval system. Moreover, we can, in theory at least, let c_{ij} reflect not only the library system's cost to provide and

transmit service to users, but can also allow for user inconvenience. This means incorporation of such considerations as distance, transportation (public, highway and street routes, parking), nearness to other facilities (shopping, offices, pedestrian centers), and general ease of use of a building such as could be placed at a specific site.

The model can also incorporate some of the aspects of the gravitational model of the location decision problems [10,26,29,48]. Here we see the entity to be located as being analogous to a body of matter, reacting to gravitational forces acting upon it and coming to an equilibrium state. The forces are proportional to the number of demands from the various users, and are inversely proportional to the distances between the users and the proposed sites for the entity. The library location model allows for this with the c_{ij} parameters being costs, a detriment, so that the inverse aspects of distance, when c_{ij} is proportional to the distance between site i and user location j, are reflected in the model. Moreover, the constraints on demand satisfaction, as we shall see, reflect the effects of the pull of the users.

In addition, f_i should reflect political as well as economical considerations. Thus the idea of an effective retrieval system is evidenced.

The model also deals with the inputs of the library as a service organization, which includes the needs of the users, the goals of the servicer, and the controls of the funder. The minimization of total cost can reflect user needs and deal with the professional goal of provision of service to the client. It can do this most directly by minimizing user inconvenience and by freeing limited capital for better and more efficient uses such as increased service. We shall also see that the constraint requiring that user demands must be satisfied by the provision of library service also reflects these service organization inputs. The idea of limited capital that must be used judiciously as well as the idea that some sites may be more expensive due to political considerations as well as economics, reflects on the input of funder controls.

Since minimizing the function(cx)yields the same result as maximizing the function (-c)x, with respect to the variable x for some constant c, let us rewrite our objective function as

(1a) Max: $\sum_{j=1}^{n}\sum_{i=1}^{m}(-c_{ij})x_{ij} + \sum_{i=1}^{m_1}(-f_i)z_i$
(x,z)

This implies that a negative cost is a worth and that the total worth is to be maximized. The expression "Max $_{(x,z)}$ " implies a maximization of the argument that follows it with respect to the decision variables x (x_{ij}) and z (z_i).

The Constraints

We cannot merely consider the objective function in isolation, since doing so would imply closing our doors, building and locating no new libraries, and transmitting no service. This means a total cost of zero but it violates the whole idea of what a library system is and does. Hence we must clearly impose restraints on the policies open to us.

First of all, we cannot transmit more service from site i ($= \sum_{j=1}^{n} x_{ij}$) than a library at site i has the capacity to provide. Letting M_i = the maximum capacity in units of service that would be available if a library were placed at site i, we have

(2) $\sum_{j=1}^{n} x_{ij} \leq z_i M_i \qquad \forall i = 1, 2, \ldots m.$

Note that the symbol "$\forall i = 1, 2, \ldots m$" implies "for all" or "for every i, i = 1, 2,... m." Letting $y_i = z_i M_i - \sum_{j=1}^{n} x_{ij}$ = the capacity at site i not transmitted to users at the various location, i.e., the excess service left unused and undemanded, we can rewrite the constraint as follows,

(2a) $\sum_{j=1}^{n} x_{ij} - z_i M_i + y_i = 0 \qquad \forall i = 1, 2, \ldots m.$

Second, the demand for service at user location j must be satisfied. Letting d_j = the number of service units required to minimally satisfy the demand of the users at location j, and noting that $\sum_{i=1}^{m} x_{ij}$ = the total number of service units transmitted to user location j from the various sites, we have

(3) $\sum_{i=1}^{m} x_{ij} \geq d_j \qquad \forall j = 1, 2, \ldots n.$

Letting $y^o_j = \sum_{i=1}^{m} x_{ij} - d_j$ = the excess service, over that minimally demanded, provided to the users at location j, we can rewrite the constraint as follows

(3a) $\sum_{i=1}^{m} x_{ij} - y^o_j = d_j \qquad \forall j = 1, 2, \ldots n;$

or

(3b) $\sum_{i=1}^{m} (-x_{ij}) + y^o_j = (-d_j) \qquad \forall j = 1, 2, \ldots n.$

By the definition of our location problem, we can build at site i no more than one library, and that one would have a fixed capacity, if

built, of M_i service units. Hence we have

(4) $z_i \leq 1 \qquad \forall_i = 1, 2, \ldots m.$

Letting y_i^{oo} = 1 - z_i = the number of libraries not placed at site i = the complement of z_i (y_i = 0 when z_i = 1 and y_i = 1 when z_i = 0), we can rewrite (4) as

(4a) $z_i + y_i^{oo} = 1 \qquad \forall_i = 1, 2, \ldots n.$

The need for this constraint, in this form, will be seen later. However, we note that if z_i is a zero-one variable, so is y_i^{oo} $\forall$i.

By the definition of the decision and slack variables, they are all non-negative. It makes no sense to consider negative service; hence we have

(5) $x_{ij} \geq 0 \quad \forall_i \forall_j; \quad z_i \geq 0 \quad \forall_i; \quad y_i \geq 0 \quad \forall_i; \quad y_j^{o} \geq 0 \quad \forall_j;$
$y_i^{oo} \geq 0 \quad \forall_i.$

Moreover, we have, again by the definition of the decision variable z_i,

(6) z_i integer $\forall_i = 1, 2, \ldots n.$

Relations (4a), (5), and (6) imply that z_i = 0 or 1 $\forall_i$ as originally defined.

The [illegible] y_i, y_j, and y_i^{oo} variables are called slack variables, taking up the excess capacity or demand or empty sites, so that the constraints can be written as equalities. They are in fact a type of nonnegative decision variable that can be determined by a specification of the values for the regular decision variables.

We shall see that the final form of the constraints, and of the objective function, will be quite useful and will lend to several interesting implications for librarians. Moreover, we note that by combining (2) and (3) and summing over all sites and all user locations, we have

(7) $\sum_{j=1}^{n} d_j \leq \sum_{j=1}^{n} \sum_{i=1}^{m} x_{ij} = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} \leq \sum_{i=1}^{m} z_i M_i$

This implies that total capacity, $\sum_{i=1}^{m} z_i M_i$, must equal or exceed the total service transmitted to users, $\sum_{j=1}^{n} \sum_{i=1}^{m} x_{ij}$, and hence must equal or exceed the total demand, $\sum_{j=1}^{n} d_j$. If $\sum_{i=1}^{m} M_i < \sum_{j=1}^{n} d_j$, there are no feasible solutions since no location decision policy will satisfy the minimal demand. Finally, if [illegible] the

M_i and d_j parameters are integers, the x_{ij}, v_i, and y_j^o decision and slack variables will also be integers [18].

The Model

Consider now the total library location model, which is a mixed zero-one linear programming problem:

(1a) $\underset{(x,y)}{\text{Max}} \sum_{j=1}^{n} \sum_{i=1}^{m} (-c_{ij}) x_{ij} + \sum_{i=1}^{m} (-f_i) z_i$

subject to

(2a) $\sum_{j=1}^{n} x_{ij} - M_i z_i + y_i = 0 \quad \forall i$

(3b) $\sum_{i=1}^{m} (-x_{ij}) + y_j^o = -d_j \quad \forall j$

(4a) $z_i + y_i^{oo} = 1 \quad \forall i$

(5) $x_{ij} \geq 0 \ \forall i,j; \quad z_i \geq 0 \ \forall i; \quad y_i \geq 0 \ \forall i; \quad y_j^o \geq 0 \ \forall j;$
$y_i^{oo} \geq 0 \ \forall i.$

(6) z_i integer $\forall i$.

The model, as exhibited in formulas (1), (2a), (3b), (4a), (5), and (6), represents a decision-making framework for the location decision problem in a library system. By a programming problem we mean a problem involving constrained optimization. The linear problem means that all expressions are linear in terms of the decision variables, and by mixed we mean that some but not all of the decision variables are constrained as integers. The decision variables (x , z) represent the various alternative selection policies in terms of which sites to use and to what extent. The objective function (16) allows us to evaluate each alternative and select that policy that best suits our purposes.

Thus we are trying to maximize worth (negative cost) (1a) through an optimal location policy that must not violate the site capacity or availability constraints (2a), the minimum user demand constraints (3b), the constraints on only one library with fixed capacity per site constraints (4a), the nonnegativity constraints (5), and the integrality constraints (6).

The library location model is based on the analogous problem of industrial warehouse location [5, 6, 9, 11, 15, 16, 19, 34, 54, 55, 56, 57, 59, 63, 64].

The location model is really an extension of the transportation problem model [18], adding the problem of site determination. In fact, if we specify values for the z_i variables, thereby specifying a particular site location policy, we are left with the transportation model, generated from the original location model [5]. The transportation model deals with the problem of determining how much service a particular site will transmit to a particular user location, i.e., he specification of the values of the x_{ij} variables. Nance [42] also uses some extensions of the transportation model to analyze library networks; so that the use of such an approach to library problems does have a precedent.

While there are obviously some similarities, there are also many contrasts. The idea of a unit of output, the concept of fixed demand, and the idea of fixed capacity all seem reasonable for warehouse situations dealing with products in wholesale markets. A library or information system is, however, more of a retail outfit with demand being partially determined by location. Moreover, the definition of a standard unit of service, while an intriguing idea with no conceptual problems, is not yet a reality in the library situation; there are too many factors such as detail and quality of service to be considered to make the unit output concept totally realistic. In addition, the library system is a service organization, often without the usual profit motives of industry. Hence the goals of a library may seem more suitable to the maximization of service rather than of profit or the minimization of cost.

In addition, Pred [47], for one, implies that optimization models do not accurately describe the situation that actually occurs during location decision-making. He feels that the assumptions of perfect information and of the ability to use such information do not hold in the real world.

Finally, it is not evident that the data needed for any specific library situation can be easily obtained. This is especially true of the unit transport costs if one is to include considerations of user inconvenience as well as distance.

However, there are some interesting implications of the model for decision makers. First of all, the process of considering such a model for use in a library location problem may allow some librarians to think of better methods to handle their specific situation. Secondly, attempting to obtain data for such a model may provide for better management information systems. In addition, solutions generated from a model may provide policy guidelines to a decision maker. Finally, we shall see that many relevant concepts can be derived from some of the mathematical manipulations that are performed on the model.

An Extension of the Model

Let us now consider a reformulation of the model to extend it to be more in line with Nance's work on library networks[42,43,44]. We note that Nance is concerned with interlibrary communication networks while we have been dealing with problems of library and information center location. Thus, we must consider user locations since our focus has been on the optimal delivery patterns of service to the clients. With this reformulation we can consider both interlibrary communication and transmission of service to users.

We must first revise slightly our notation so that it incorporates the concept of a relay station. A relay station, should it be placed at a given site, would transmit messages or serivces between various nodes of the network. Thus we must locate both libraries and relay stations. Recall that L = the set of m potential library sites and N = the set of n user locations. Let G = the set of g potential relay station sites. Each subscript i and j now will represent any node of this library network, where a node can be any library site, relay station site, or user location. Thus a node is any source, destination, or intermediate relay in the network. If a subscript is in the range (1,2,...,m), it represents a potentail library site; if it is in the range (m+1,m+2,...,m+g), it represents a potential relay site; and if it is in the range (m+g+1,M+g+2,...,m+g+n), it represents a user location. Finally, we ass a third subscript, k, to represent the type of communication channel or delivery vehicle, such as telephone, teletype, personal delivery, etc.[42], between two nodes. Note that the first subscript indicates the node source of the message, the second subscript indicates the node destination of the message, and the third subscript indicates the communication channel over which the message is to be carried. Thus, we how have

x_{ijk} = the number of service units or messages transmitted from node i to node j over channel k,

z_{ik} = a binary-valued decision variable, set equal to one if a channel of type k

is located at site i, and equal to zero if not,

c_{ijk} = the expected transport cost to deliver one unit of service or one message from node i to node j over communication channel k,

f_{ik} = the expected site cost to locate a channel of type k at site i,

M_{ik} = the maximum capacity of a channel of type k in terms of the units of service or of messages that would be transmitted if a facility would be located at site i,

d_{jk} = the number of service units or messages required to minimally satisfy the need or demand at node j transmitted over a channel of type k,

r(ij) = the number of different types of communication channels possible between nodes i and j,

r(i) = the number of different types of communication channels that can possibly emanate from node i,

and

e_{ijk} = the maximum capacity of a channel of type k to transmit service or messages from node i to node j.

To be consistent and realistic, we must let $c_{ijk} = \infty$ and $e_{ijk} = 0$ $\forall k, \forall i=j$, so that no node can give service or transmit messages to itself. We must also let $f_{ik} = \infty$ $\forall k, \forall i=m+g+1,\ldots,m+g+n$, so that we do not locate a facility at a user location but only at potential library and relay station sites; let $M_{ik} = 0$ $\forall k, \forall i = m+1,\ldots,m+g+n$, so that neither users nor relays have any capacity; also let $d_{ik} = 0$ $\forall k, \forall j = m+1,\ldots,m+g$, so that no relay can generate its own demand. This allows libraries to intercommunicate with messages as well as to give service to users.

The model now is a transshipment formulation [14], and has become the following

1b) $\underset{(x,z)}{\text{Max}} \sum_{k=1}^{m+n+g} \sum_{j=1}^{m+n+g} \sum_{i=1}^{\lambda(j)} (-c_{ijk}) x_{ijk} + \sum_{i=1}^{m+n+g} \sum_{k=1}^{\lambda(i)} (-f_{ik}) z_{ik}$

s.t.

2b) $\sum_{j=1}^{m+n+g} x_{ijk} - M_{ik} z_{ik} + y_{ik} = 0 \quad \forall i,k$

2c) $\sum_{d=1}^{m+n+g} (x_{ijk} - x_{jik}) = 0 \quad \forall k, \forall i = m+1, \ldots, m+g$

3c) $\sum_{i=1}^{m+n+g} (-x_{ijk}) + y^{o}_{jk} = (-d_{jk}) \quad \forall j,k$

4b) $z_{ik} + y^{oo}_{ik} = 1 \quad \forall i,k$

5a) $x_{ijk} \geqq 0 \ \forall i,j,k; \quad z_{ik} \geqq 0 \ \forall i,k; \quad y_{ik} \geqq 0 \ \forall i,k; \quad y^{o}_{jk} \geqq 0 \ \forall j,k; \quad y^{oo}_{ik} \geqq 0 \ \forall i,k$

6a) z_{ik} integer $\forall i,k$

6b) $x_{ijk} \leqq e_{ijk} \quad \forall i,j,k$

Note again that we have added to our model the concept of messages between libraries, of relay stations, and of various types of communication channels. Moreover, each channel has a limited capacity for service or message transmission. The relay stations must be located just as libraries must, and the quantity of transmission from node to node over each channel must be determined for a given cost, capacity, and demand situation. We have added the constraints 6b) to indicate the limited capacity of the communication channels between nodes. The constraints 2c) imply that there is no net capacity or demand at a relay station. Thus, our model is now closely related to Nance's . If the originating node i and the destination node j are in the range (1,2,...,m), the transmission is a message between libraries; otherwise, the transmission could be either a message or a unit of service. The only way to distinguish between messages and service is by the various communication channels employed. One can control the flow of either commodity through the specification of the e_{ijk}, c_{ijk}, M_{ik}, and d_{jk} parameters. For example, if we wish to restrict the flow of messages from libraries to relays, let $c_{ijk} =$ and $e_{ijk} = 0$ $\forall i=1,\ldots,m$, $\forall k$ representing the channels of message flow from a library to a relay. Finally note that the major differences, besides slight notational discrepancies, between our model and Nance's is that we do not sum over the various channels in constraints 2b), 2c) and 3c). Thus our model is a bit more general in that we

can generate Nance's model from ours by such summation. The advantage here is that we can add more detail about the effects of channel location to aid inbetter location decisions.

The one major disadvantage both models have is that neither allows for the decision problems of routing. That is, we can optimally locate facilities and determine the best levels of transmission between nodes over which channels of communications, but we cannot determine which route is best for a specific message under a specific set of conditions of cost and capacity. This is then one logical next item for further research. However, we have combined interlibrary communication and service transmission into one global model. Moreover, we can include extras such as interuser communication with only slight changes in the basic parameters of the model.

The Dual Problem

Consider now another mathematical optimization model to be called the dual problem. This model will be somewhat similar to our original library location model of page 8, henceforth to be called the primal problem.

A general theory of dual programming problems is developed and discussed in [4]. Using this theory, the dual problem for the library location model is given below. The use of the same parameters c_{ij}, f_i, M_i, and d_j, for all the values of i and j and the use of the same z_i and slack y_i^{oo} variables for all values of i is noted.

(7) $\underset{(z)}{Max} \; \underset{(u,u^o,u^{oo})}{Min} \; -\sum_{j=1}^{n} u_j^o d_j - \sum_{i=1}^{m} v_i z_i + \sum_{i=1}^{m} u_i^{oo}$

subject to

(8) $u_i - u_j^o - v_{ij} \geq -c_{ij} \quad \forall i,j$

(9) $-M_i u_i + u_i^{oo} - v_i \geq -f_i \quad \forall i$

(10) $u_i \geq 0 \; \forall i$; $u_j^o \geq 0 \; \forall j$; $u_i^{oo} \geq 0 \; \forall i$; $v_{ij} \geq 0 \; \forall i,j$; $y_i^{oo} \geq 0 \; \forall i$; $z_i \geq 0 \; \forall i$

(11) v_i unrestricted in sign $\forall i$.

(12) $z_i + y_i^{oo} = 1 \quad \forall i$

(13) z_i integer $\forall i$.

Dual Variable Definitions

We now need to define the dual variables u_i, u_j^o, and u_i^{oo}, and the dual slack variables v_i and v_{ij}. We have

u_i = the per unit "worth" of the capacity of a library at site i, and the amount by which the total cost function (1) could have been lowered had we increased the site i capacity to $(M_i + 1)$ when a library is placed at site i ($z_i = 1$),

u_j^o = the per unit "worth" of the demand at user location j, and the amount by which the total cost function (1) could have been lowered had the demand at location j been decreased to $(d_j - 1)$,

u_i^{oo} = the "worth" associated with being able to place at maximum only one library, with fixed capacity of M_i units of service, at site i, and

the amount by which the total cost function (1) could have been lowered by allowing the possibility of another library to be located at site i in addition to the original library, both with capacity M_i.

These dual variables and the slack dual variables, v_i and v_{ij}, as well as the primal variables z_i and primal slack variables v_i^{oo}, are the decision variables for the dual problem. The dual model then is useful only as far as it yields further insights into the library location problem.

These dual variables, u_i, u_j^o, and u_j^{oo}, really represent, at optimum, the worth of the opportunity to change the total optimal cost of operation (1). This change occurs by changing the availability of service (M_i), the demand for service (d_j), or the potential number of libraries that can be placed at one site. These opportunities can be viewed as a worth, a positive and good thing, as opposed to a cost; although they are sometimes referred to as opportunity costs. Moreover, reduction of the total cost function (1), which we wish to minimize, is the same as increasing the total worth function (1a), which we want to maximize, since negative cost is considered as a worth and negative worth is considered as a cost. Thus, $-u_i$ or $-u_j$ is seen as a cost.

Finally, we note the concern of the dual decision variables u_i and u_i^{oo} with the total capacity for service at site i and the concern of u_j^o with the satisfaction of user demand at location j.

Dual Constraint Interpretation

The dual constraints can be interpreted in terms of what the optimal primal solution really means. We shall see that there are several implications of the dual constraints for a librarian. Moreover, we shall see what the dual slack variables are in terms of the overall picture.

(8) $u_i - u_j^o - v_{ij} = -c_{ij} \qquad \forall i,j$

This constraint can be rewritten as

21 $v_{ij} = c_{ij} + u_i - u_j^o$, so that we can define v_{ij} as the sum of the

unit cost to ship a service unit from site i to user location j plus the opportunity cost associated with the capacity at site i minus the opportunity cost associated with the demand at location j. It is the net cost associated with the shipment of one unit of service over the route from site i to location j. Using this route to ship one service unit, we in effect decrease the resulting capacity at site i by one unit and decrease the resulting demand at user location j by one unit, which yields a resulting worth of $u_j^o - u_i$, or a cost of $-(u_j - u_i) = u_i - u_j^o$, in addition to incurring the usual unit transportation cost c_{ij}.

(10) $v_{ij} \geqq 0 \quad \forall_{ij}$ further implies that this net cost is nonnegative. This can be rewritten as $c_{ij} \geqq u_j^o - u_i \quad \forall_{i,j}$ which implies that the transport cost from site i to location j can be no smaller than the net worth of having the capacity to satisfy user demands. Hence, as Balas [4] points out, no activity will yield a positive profit.

(9) $-M_i u_i - v_i + u_i^{oo} = -f_i \quad \forall_i$.

This constraint can be rewritten as $v_i = f_i + u_i^{oo} - M_i u_i$, defining v_i as the sum of the costs to place a library at site i plus the opportunity cost associated with being able to only place one library at site i minus the total opportunity cost associated with the capacity of M_i service units at site i. Hence, v_i represents the net cost of locating a library at site i.

(11) v_i unconstrained in sign $\forall_i$, implies that one cannot tell if a library at site i will be profitable or not.

The constraints on z_i and y_i^{oo} (10, I2, 13) are the same as in the primal problem. We must also note the nonnegativity (10) of the dual variables.

The Dual Objective Function

The dual objective function,

(7) $\underset{(z)}{\text{Max}} \quad \underset{(u,u^o,u^{oo})}{\text{Min}} \quad -\sum_{j=1}^{n} u_j^o d_j - \sum_{i=1}^{m} v_i z_i + \sum_{i=1}^{m} u_i^{oo}$

is to be maximized with respect to the z_i variables while being simultaneous-

ly minimized with respect to the u_i, u_j^c, and u_i^{oo} variables. We note that these variables are the dual decision variables. The expression $-\sum_{j=1}^{n} u_j^c d_j$, representing the negation of the total worths associated with the satisfaction of user demand at the various user locations, minimized over the u_j^c variables, implying in reality maximization of the total worth of user demand satisfaction. This of course relates directly to the library as a service organization. The expression $\sum_{i=1}^{m} u_i^{oo}$ represents the total cost of being able to place only one library of fixed capacity at any site, and is minimized with respect to the u_i^{oo} variables. The expression $-\sum_{i=1}^{m} v_i z_i = \sum_{i=1}^{m} (M_i u_i - f_i - u_i^{oo}) z_i$ represents the net worths of providing a capacity of M_i service units at site i summed over all the sites; these worths being incurred only when a library is placed at site i, i.e., $z_i=1$. This total net worth is minimized with respect to the u_i and u_i^{oo} variables and is maximized with respect to the z_i variables. The negative coefficient assigned to the u_i^{oo} variables really implies a maximization of net worth with respect to u_i^{oo}. Note that we can combine $\sum_{i=1}^{m} u_i^{oo} + \sum_{i=1}^{m} (M_i u_i - f_i - u_i^{oo}) z_i$ to yield $\sum_{i=1}^{m} (M_i u_i - f_i) z_i + \sum_{i=1}^{m} u_i^{oo} (1 - z_i)$ $= \sum_{i=1}^{m} (M_i u_i - f_i) z_i + \sum_{i=1}^{m} u_i^{oo} y_i^{oo}$ The first term is the net worth of physically placing libraries at the various sites, the second term is the cost of not placing a library at the various sites. This second term will be seen to be equal to zero at optimum.

Properties of the Dual Model

We shall point out several properties of the dual model and discuss their implications.

Involution

The dual of the original dual problem yields the original primal problem. Moreover, we see an interesting relationship between the primal and dual variables:

x_{ij} continuous $\longleftrightarrow$ $v_{ij} \geq 0$ $\forall i,j$;

z_i integer $\longleftrightarrow$ v_i unconstrained in sign $\forall i$;

$y_i \geq 0$, $y_j^o \geq 0$, $y_i^{oo} \geq 0$ $\longleftrightarrow$ u_i, u_j^o, u_i^{oo} continuous $\forall i, \forall j$.

Bounds

Let $\bar{X}$ = the set of all (x_{ij}, z_i) variables such that the conditions

$x_{ij} \geq 0$, $z_i \geq 0$, $z_i \leq 1$, z_i integer,

are met $\forall i,j$. This set $\bar{X}$ is the set of all feasible solutions to the primal problem of page 8; that is, the set of all (x_{ij}, z_i) variables satisfying constraints (2a), (3b), (4a), (5), and (6). Let $\bar{U}$ = the set of all (u_i, u_j^o, u_i^{oo}) variables such that the conditions $u_i \geq 0$, $u_j^o \geq 0$, $u_i^{oo} \geq 0$, $u_i - u_j \geq c_{ij}$, $-M_i u_i + u_i^{oo} - v_i = F_i$, v_i unconstrained in sign $\forall i,j$.

This set $\bar{U}$ is the set of all feasible solutions to the dual problem of page 10; that is, the set of all (u_i, u_j^o, u_i^{oo}) variables satisfying constraints (8), (9), (10), and (11). Let $\bar{Z}$ = the set of z_i variables such that the conditions $z_i \leq 1$, $z_i \geq 0$, z_i integer are met $\forall i$. The set $\bar{Z}$ is the set of all z_i variables satisfying the constraints (10), (12), and (13). We shall also define

$$Z = \sum_{j=1}^{n} \sum_{i=1}^{m} (-c_{ij}) x_{ij} + \sum_{i=1}^{m} (-f_i) z_i = \text{the primal objective function (1a)},$$

and

$$W = -\sum_{j=1}^{n} u_j^c d_j - \sum_{i=1}^{m} v_i z_i + \sum_{i=1}^{m} u_i^{oo} = \text{the dual objective function (7)}.$$

Finally, $Z^* = \max_{\bar{X}} Z$ = the optimal primal objective function, and $W^* = \max_{\bar{Z}} \min_{\bar{U}} W$ = the optimal dual objective function. If there exist primal feasible solutions ($\bar{X}$ is nonempty) and dual feasible solutions with $v_i \geq 0$ $\forall i$, then there exists a feasible primal and feasible dual solution such that

$$Z \leq -\sum_{j=1}^{n} u_j^o d_j + \sum_{i=1}^{m} u_i^{oo}.$$

If no feasible solutions exist for the primal problem $Z^* = -\infty$; and if no feasible solutions exist for the dual problem, $W^* = +\infty$. If a feasible solution exists for either or both problems, $Z^* \leq W^*$. Thus, the optimal dual objective function is a upper bound on the optimal primal objective function.

Complementary Slackness

If we have a feasible and optimal primal solution (x^*, z^*), there exists a feasible dual solution $(u^*, u^{\circ *}, u^{\circ\circ *})$ such that $\sum_{i=1}^{m} u_i^* y_i^* + \sum_{j=1}^{n} u_j^{\circ *} y_j^{\circ *} + \sum_{i=1}^{m} u_i^{\circ\circ *} y_i^{\circ\circ *} = 0$. The slack variables from both the primal and dual problems can be computed from their respective decision variable values. The nonnegativity constraints (5) and (10) give us the fact that $u_i^* y_i^* = 0 \quad \forall i$, $u_i^{\circ\circ *} y_i^{\circ\circ *} = 0 \quad \forall i$, $u_j^{\circ *} y_j^{\circ *} = 0 \quad \forall j$. Moreover, we have $\sum_{j=1}^{n}\sum_{i=1}^{m} x_{ij}^* v_{ij}^* = 0$, which implies that $x_{ij}^* v_{ij}^* = 0 \quad \forall i,j$. Thus, for the continuous optimal primal decision variables x_{ij}^*, a positive x_{ij}^* implies a zero dual slack variable v_{ij}^*, and vice versa. For the dual decision variables, a positive value here implies a zero value for the corresponding optimal primal slack variables, and vice versa. Excess capacity or oversupply in the provision of demand implies that the worth of change (increasing capacity or lessening demand) is zero. Moreover, one ships from site i to location j ($x_{ij} > 0$) only when the net cost (v_{ij}) is equal to zero.

Saddlepoints

If the primal problem has an optimal solution, (x^*, z^*), and if the dual problem has as its optimal solution $(u^*, u^{\circ *}, u^{\circ\circ *})$, $z^* = w^*$. Thus, the optimal total worth of locating libraries and transmitting service is equal to the imputed (opportunity) cost or value of the resource commodities (capacity and demand).

Defining the function $F(x,z,u,u^{\circ},u^{\circ\circ})$ as follows

(14) $$F(x,z,u,u^{\circ},u) = \sum_{j=1}^{n}\sum_{i=1}^{m}(-c_{ij})x_{ij} + \sum_{i=1}^{m}(-F_i)z_i - \sum_{j=1}^{n} u_j^{\circ} d_j + \sum_{i=1}^{m} u_i^{\circ\circ} - \sum_{i=1}^{m} u_i\left(\sum_{j=1}^{n} x_{ij} - M_i z_i\right)$$

$$- \sum_{j=1}^{n} u_j^{\circ}\left(\sum_{i=1}^{m}(-x_{ij})\right) - \sum_{i=1}^{m} u_i^{\circ\circ} z_i$$

$$= \sum_{j=1}^{n}\sum_{i=1}^{m}(-c_{ij} - u_i + u_j^{\circ})x_{ij} + \sum_{i=1}^{m}(-F_i + M_i u_i - u_i^{\circ\circ})z_i + \sum_{i=1}^{m} u_i^{\circ\circ} - \sum_{j=1}^{n} u_j^{\circ} d_j$$

$$= \sum_{j=1}^{n}\sum_{i=1}^{m}(-v_{ij})x_{ij} + \sum_{i=1}^{m}(-v_i)z_i - \sum_{j=1}^{n} u_j^{\circ} d_j + \sum_{i=1}^{m} u_i^{\circ\circ}$$

$$= \sum_{j=1}^{n}\sum_{i=1}^{m}(-v_{ij})x_{ij} + w$$

we have

$$F(\chi, Z, u^*, u^{o*}, u^{oo*}) \leqq F(\chi^*, Z^*, u, u^{o*}, u^{oo*}) \leqq F(\chi^*, Z^*, u, u^o, u^{oo})$$

for all feasible sets of primal and dual variables.

This implies that the value of F for the optimal primal and dual solution can be decreased by using nonoptimal feasible primal decision variables. Hence, we are maximizing F with respect to the primal, and minimizing F with respect to the dual decision variables. We note that, because of complementary slackness, $F(\chi^*, Z^*, u^*, u^{o*}, u^{oo*}) = W^* = Z^*$.

The graph depicted below will explain why we call this phenomenon a saddle-point solution.

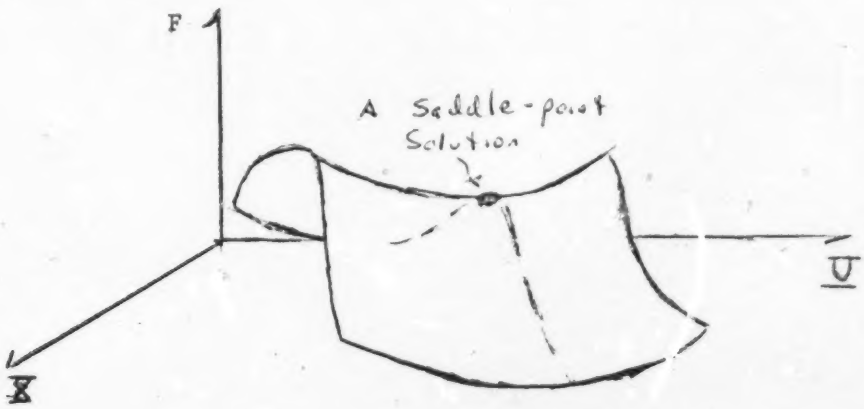

Figure 1. A Saddle-point Solution

Existence

One and only one of the following conditions holds:

1. $Z^* = +\infty$, $W^* = +\infty$, no feasible dual solution with $v_i \geqslant 0$ Vi exists, primal solution is unbounded
2. $Z^* = -\infty$, $W^* = -\infty$, no feasible primal solution exists, dual solution is unbounded
3. Z* and W* are undefined, no feasible primal solution or feasible dual solution with $v_i \geqslant 0$ Vi exist
4. $Z^* = -\infty$, $W^* = +\infty$, no feasible primal or feasible dual solutions exist
5. Z* = W*, Z* finite, both primal and dual problems have feasible, and optimal, solutions

The case where $Z^* = +\infty$, $W^* = -\infty$ cannot exist. Also, we define an unbounded problem as one in which one can find better and better solutions with no limit so that one can reach ∞ infinity as the value for the objective function.

Inconsistency

The existence of the primal decision variables z_i in the dual objective function is somewhat of an oddity and must be explored further. The appearance of these primal variables in the dual problem is, according to Balas [4] who is responsible for duality theory in discrete or integer programming problems, due to the integrality constraints on the z_i variables. If we could drop the integrality constraints, a situation to be explored below, we could omit the expression $-\sum_{i=1}^{m} v_i z_i$ from the dual objective function which would remove the problem of having to maximize the function with respect to the z_i variables. With the integrality constraints, we must find a feasible solution to the primal problem that makes the primal objective function as large as possible.

The dual constraints of (9), rewritten for the case where $v_i \geq 0$ $\forall i$ as

(9b) $\quad -M_i u_i + u_i^{oo} \geqslant (-f_i) \quad \forall i,$

are in fact "relaxed" by the introduction of the dual slack variables v_i which are unconstrained in sign. This "relaxation" or "gap" occurs when the constraint is "violated", that is, when $v_i < 0$. This "violation" is due to the integrality of z_i which allows v_i to be unconstrained in sign so that v_i can be negative. Hence $(-v_i)$ represents the amount by which the constraint is potentially "violated" or "relaxed".

We weight the variable $(-v_i)$ by the variable z_i and include the sum $(-\sum v_i z_i)$ in the dual objective function, maximizing the function with respect to the z_i variables and minimizing it with respect to the v_i variables.

Linear Programming Models and Their Implications

Consider the following problem,

(1b) $\quad \max_{\bar{x}} \sum_{j=1}^{n} \sum_{i=1}^{m} (-c_{ij}) x_{ij} + \sum_{i=1}^{m} (-f_i + s_i^{\circ}) z_i$

subject to

(2a) $\quad \sum_{j=1}^{n} x_{ij} - M_i z_i + y_{i\circ} = 0 \quad \forall i$

(3b) $\quad \sum_{i=1}^{m} (-x_{ij}) + y_{\circ j}^{\circ} = -d_j \quad \forall j$

(4a) $z_i^{**} + y_i^{**} = 1 \quad \forall i$

(5) $x_{ij} \geq 0 \ \forall i,j$; $y_i \geq 0 \ \forall i$; $y_j^{\circ} \geq 0 \ \forall j$; $y_i^{\circ\circ} \geq 0 \ \forall i$; $z_i \geq 0 \ \forall i$.

Note that this is the primal problem with two changes; we have deleted the integrality constraints (6) on the z_i variables (hence the problem is strictly linear) and have changed the coefficients of the z_i variables in the objective function from $(-f_i)$ to $(-f_i + s_i)$. If we let $s_i = \begin{cases} v_i^* & \text{if } z_i^* > 0 \\ \min[0, v_i^*] & \text{if } z_i^* = 0 \end{cases}$, from the solution to the original dual problem, we can show that the optimal primal solution to the original primal problem is the optimal solution to the linear programming problem. The dual problem of the linear programming model is

(7a) $\underset{V}{\text{Min}} \; -\sum_{j=1}^{n} u_j^{\circ} d_j + \sum_{i=1}^{m} u_i^{\circ\circ}$

Subject to

(8) $u_i - u_j^{\circ} - v_{ij} = -c_{ij} \quad \forall i,j$

(9a) $-M_i u_i + u_i^{\circ\circ} - v_i = -F_i + s_i \quad \forall i$

(10a) $u_i \geq 0 \ \forall i$; $u_j^{\circ} \geq 0 \ \forall j$; $u_i^{\circ\circ} \geq 0 \ \forall i$; $v_{ij} \geq 0 \ \forall i,j$

(11a) $v_i \geq 0 \quad \forall i$

The difference between this and the original dual is the appearance of s_i in the constraints of (9a), the disappearance of all references to the z_i and $y_i^{\circ\circ}$ variables, and the nonnegativity of the variable v_i (11a). Moreover, in the linear programming problem, $v_i^* z_i^* = 0 \quad \forall i$, implying that no activity occurs unless there is not a positive net cost associated with that activity. We recall from page 12 that $v_i^* = -M_i u_i^* + u_i^{\circ\circ *} + f_i$. Thus we have the fact that one and only one of the following cases holds:

1. $z_i^* = 0$, which means that $\sum_{j=1}^{n} x_{ij} = 0$ as no library being placed at site i implies that no service can eminate from there, and $s_i \leq 0$, which means that a penalty is incurred at an unused site.

2. $z_i^* = 1$ and $x_{ij} = 0$, which means that a library is located at site i but no service eminates from it which can occur only if $f_i = 0$, and $s_i = f_i + u_i^{\circ\circ *} = u_i^{\circ\circ *}$, which means that a subsidy is incurred at a free site.

3. $z_i^* = 1$ and $0 < \sum_{j=1}^{n} x_{ij} < M_i$, which means that a library is located at site i and is used but not to capacity, and $s_i = f_i + u_i^{\circ\circ *}$, which means that a subsidy is incurred at a site used to below capacity.

4. $z_i^* = 1$ and $\sum_{j=1}^{n} x_{ij} = M_i$, which means that a library is located at site i and

and is used to capacity, and $s_i = f_i + u_i^{oo}* - M_i u_i \lesssim f_i + u_i^{oo}*$, which means that a smaller subsidy or even a penalty is incurred at a site used to capacity.

The meaning of s_i is that it is a penalty ($s_i \leq 0$) or a subsidy ($s_i > 0$), due to the integrality of z_i in the original problem. Thus, (9a) implies that no activity (library location) can yield a positive profit.

In cases 1, 2, and 4, the higher the penalty, or the lower the subsidy, the better the site is. Also note ... For case 3, s_i is restricted to being equal to the upper bound on s_i in case 4, which means that a library that is used but not to capacity must remain open (z_i cannot be rounded down to zero). If $z_i* = 0$, as in case 1, the site is poor, and so we pay a penalty in order to not locate a library there. Finally, if $f_i = 0$, as in case 2, we may locate libraries there at will as it is a free activity.

The implication is that we can simplify the original problem. Instead of either building a library of fixed capacity of site i (M_I) or nothing at all, let us be more flexible in our approach. Letting M_i represent the maximum capacity for service possible at site i, consider the possibility of building a library at site i with a capacity anywhere in the range from zero to M_I. This can be modeled as the original primal problem of

page 10 with the integrality of the z_i variables constraints (6) deleted. This means that the z_i now represents the fraction of the total possible capacity, M_i, that will be placed at site i, so that $z_i M_i$ is the capacity placed at site i. Thus, f_i is now the cost per service unit of placing service capacity at site i. We now have the linear programming problem of page 17, with $s_i = 0 \quad \forall i$; this is so because there is no penalty or subsidy since there are no integrality constraints.

However, there is one important drawback to be considered. In setting up the library location model in this fashion; we no longer allow for a fixed cost of location at a specific site, a cost independent of the capacity placed at the site. In order to compensate for this, we would have to add a factor, g_i, to the total cost function, defined as

$$g_i = \begin{cases} 0 & \text{if } z_i = 0 \\ g_i^0 & \text{if } z_i > 0 \end{cases},$$

where g_i^0 = the fixed cost of locating a library of positive capacity at site i. This becomes extremely complex to represent without 0-1 integer variables, which brings us right back to where we started. Thus, unless we assume that $g_i^0 = 0 \quad \forall i$, we must use zero-one integer variables as in the original primal problem.

Summary and Conclusions

In this paper we have presented a mixed zero-one linear programming model of the library or information center location problem. The objective function has been shown to reflect the goals of the library as an information retrieval system and as a service organization. Moreover, we see the constraints on limited service capacity and requirements to at the very least minimally satisfy user demand. We use the approach of the industrial warehouse location problem with the concept of a unit of service being the crux of the model. We note both the analogies and problems of this approach to our specific problem.

The form of the objective function implies the minimization of the total cost associated with the location of libraries. We turn this around into the maximization of worth (negative cost). The key elements are the decision variables, the binary valued variables representing whether or not a library is to be placed at specific sites, and the continuous variables representing the amount of service transmitted from those sites to the various user locations. The objective function is then linear, in line with models of other library problems [33].

The constraints were formed to reflect the effects of limited capacity and of imposed demand which must be satisfied. We also note the nonnegativity of all decision variables and slacks, as well as the zero-one binary values for the z_i variables.

The general solution mechanism and dual problem are seen to yield several interesting implications for decision makers. Many of these implications relate directly to the economics of the situation, which is basically what the model and the optimization procedures are all about. We see that no activity, library location or transmission of service, can yield a positive profit or will occur if the net cost is negative. Moreover, we can see that the primal constraints generate corresponding dual variables or opportunity costs. The form of the dual, concerned with maximizing the provision of service in order to satisfy user demand, seems more in line with the goals of an information system. The dual problem is not, however, a substitute for the original primal problem. It is but another way of seeing the problem, gaining new insights from a different viewpoint of the model. In addition, we note the possibility of a linear programming approach.

The implementation of the location model to produce policy decisions in an actual environment seems to be the next required step for research.

The decision model has been developed, and it has been used and found to perform well [5,45] for other situations. We must develop means to gather the data that will yield the parameters c_{ij}, f_i, M_i, d_j for all i and j. Even more important is the definition of a unit of service that can describe all the different types and levels of library output .

Finally, the librarian must never allow this model, or any other, to replace him as the ultimate decision-maker. This implies that the responsibility for decisions must rest with the person, not the tool. The model is just that, a tool, to be used as a guide and an aid to the decision-maker. With this in mind, it is our hope that general uses may be found for the model in relation to the location decision in a library system, and that perhaps the approach may also be applied to other problems.

BIBLIOGRAPHY

1. Armour, G. C. and E. S. Buffa, "A Heuristic Algorithm and Simulation Approach to Relative Location of Facilities," Management Science, vol. 9, 1963, p. 294.

2. Backman, D. J., Retail Strategy and Structure: A Management Approach, Prentice-Hall, New Jersey, 1969, especially pp. 350-67.

3. Baker, N. R., "A Descriptive Model of Library/User/Funder Behavior in a University Environment", Drexel Library Quarterly, vol. 4, 1968, pp. 16-30.

4. Balas, E., "Duality in Discrete Programming", Reprint No. 519, Graduate School of Industrial Administration, Carnegie-Mellon University, Pittsburgh, Pa., 1970.

5. Balfin, R. L. and V. E. Unger, "An Algorithm for the Plant Location Problem", a paper presented at the 39th National Meeting of the Operations Research Society of America, Dallas, Texas, May, 1971.

6. Baumol, W. and P. Wolfe, "A Warehouse Location Problem", Operations Research, vol. 6, 1958, p. 252.

7. Becker, J. and R. Hayes, Information Storage and Retrieval: Tools, Elements, Theories, Wiley, New York, 1963.

8. Conference on Interlibrary Communications and Information Networks, Warrenton, Virginia, 1970.

9. Cooper, L., "Location-Allocation Problems", Operations Research, vol. 11, 1963, p.331.

10. Coughlin, R. E., F. Taieb, and B. H. Stevens, "Branch Library Location and the Achievement of Systems Objectives", unpublished draft, Regional Science Research Institute, Philadelphia, Pa., 1968.

11. Curry, G. L. and R. W. Skeith, "A Dynamic Programming Algorithm for Facility Location and Allocation", AIIE Transactions, vol. 1, 1969 p. 245.

12. Dei Rossi, J.A., R.S. Heiser, and N.S. King, "A Cost Analysis of Minimum Distance TV Networking for Broadcasting Medical Information", Memorandum RM-6204-NLM, Rand Corp., Santa Monica, Cal., 1970.

13. Dei Rossi, J.A., C.R. Lindholm, G.F. Mills, and G.C. Sumner, "A Telephone Access Biomedical Information Center", Memorandum RM-6205-NLM, Rand Corp., Santa Monica, Cal., 1970.

14. Duggan, M., "Library Network Analysis and Planning (LIB-NAT)", Journal of Library Automation, vol. 2, 1969, pp. 157-175.

15. Eilon, S. and D. P. Dezul, "Siting A Distribution Centre, An Analagous Computer Application", Management Science, vol. 12, 1966, p. 245.

16. Feldman, E., F. A. Lehrer, T. L. Roy, "Warehouse Location Under Continuous Economies of Scale", Management Science, vol. 12, 1966, p. 670.

17. Garrison, G., "Some Recent Public Library Branch Location Studies by City Planners", Library Quarterly, vol. 36, 1966, pp. 151-155.

18. Gass, S., Linear Programming, McGraw-Hill, New York, 1969.

19. Goldman, A. J. and C. J. Witzgall, "A Localization Theorem for Optimal Facility Placement", Transportation Science, vol. 4, 1970, pp. 406-409.

20. Greene, W. E., Plant Location Factors, Adams Press, Chicago, Illinois, 1969.

21. Greenhut, M. L., Plant Location in Theory and Practice, University of North Carolina Press, Chapel Hill, North Carolina, 1956.

22. Grundt, L., "Branch Library Inadequacies in a Typical Large City", Library Journal, vol. 90, October 1, 1965, pp. 3997-4001.

23. Hacker, H. L, "Financial Problems of the Large Library", Library Quarterly, vol. 38, 1968, pp. 41-57.

24. Haggett, P., Locational Analysis in Human Geography, St. Martin's Press, New York, 1966.

25. Hamburg, M.,R.C. Clelland, M.R.W. Bommer, L.E. Ramist, and R.M. Whitfield, "A Systems Analysis of the Library and Information Science Data System: The Research Investigation", interim report to the U.S. Office of Education, University of Pennsylvania, Philadelphia, 1970.

26. Harris, B., "Some Problems in the Theory of Intra-Urban Location", Operations Research, vol. 9, 1961, pp. 695-721.

27. Holm, B.E., "Library and Information Center Management", in Cuadra, C. (ed.), Annual Review of Information Science and Technology, Interscience Publishers, vol. 5, 1970.

28. Isard, W., Location and Space Economy, M.I.T. Press, Cambridge, Mass., 1956.

29. Isard, W., D. Bramhill, G. Carrothers, J. Cumberland, L. Moses, D. Price, and E. Schooler, Methods of Regional Analysis: An Introduction to Regional Science, The Technical Press of M.I.T. and Wiley, New York, 1960.

30. Jacobs, I., "Where to Locate a Bookstore in a City and Where Not to Locate One", Publishers Weekly, vol. 18, 1962, pp. 28-31.

31. Johnson, G. and C. Johnson, "Library Location on Oregon Campuses", PNLA Quarterly, vol. 31, 1967, pp. 146-150.

32. Korfhage, R.R., U.N. Bhat, and R.E. Nance, "Graph Models for Library Information Networks", Technical Report CP71-0013, Computer Science/Operations Research Center, Southern Methodist University, Dallas, Texas, 1971.

33. Kraft, D. H. and T. W. Hill, Jr., "A Journal Selection Model and Its Implications for Library Systems", a paper submitted to Library Quarterly.

34. Kuehn, A. and M. Hamburger, "A Heuristic Program for Locating Warehouses", Management Science, vol. 9, 1963, p. 643.

35. Leimkuhler, F.F., "Mathematical Models for Library Systems Analysis", Drexel Library Quarterly, vol. 4, 1968.

36. Leimkuhler, F. F., and A. Neville, "The Uncertain Future of the Library", The Johns Hopkins University Magazine, Fall 1967.

37. Liesener, J., Chisholm, M., Kraft, D., and Levitan, K., "Assessing the Relationships Between Resources and Services in the School Media Centers", proposal submitted to State Department of Education, School of Library and Information Services, University of Maryland, College Park, Maryland, 1971.

38. Marshall, J. U., The Location of Service Towns, University of Toronto Press, Toronto, Canada, 1969.

39. Metcalf, K., Planning Academic and Research Library Buildings, McGraw-Hill, New York, 1965.

40. Metcalf, K. P., "Why We Need to Be Investigated", College and Research Libraries, 1954.

41. Morse P., Library Effectiveness: A Systems Approach, M.I.T. Press, Cambridge, Mass., 1968.

42. Nance, R. E., "An Analytical Model of a Library Network", Journal of the American Society for Information Science, vol. 21, January-February, 1970, pp. 58-66.

43. Nance, R.E., R.R. Korfhage, and U.N. Bhat, "Information Networks:Definitions and Message Transfer Models", Technical Report CP-710011, Computer Science/ Operations Research Center, Southern Methodist University, Dallas, Texas, 1971.

44. Nance, R.E., R.E. Wickham, and M. Duggan, "A Computer System for Effective Management of a Medical Library Network: An Overview", Technical Report CP-71005, Computer Science/Operations Research Center, Southern Methodist University, Dallas, Texas, 1971.

45. Nugent, C. E., T. E. Vollman, J. Ruml, "An Experimental Comparison of Techniques for the Assignment of Facilities to Locations", Operations Research, vol. 16, 1968, p. 150.

46. Overhage, C.F., "Information Networks", in Cuadra, C. (ed.), Annual Review of Information Science and Technology, Interscience Publishers, vol. 4, 1969.

47. Pred, A., Behavior and Location Part I, Royal University of Lund, Sweden.

48. Regional Planning Council, Retail Market Alternatives in the Baltimore Region, The Technical Record, Baltimore Regional Planning Council, Baltimore, Maryland, 1967.

49. Revelle, C., D. Marks, J. Liebman, "An Analysis of Private and Public Sector Location Models", Management Science, vol. 16, 1970, p. 692.

50. Saaty, T. L., Optimization in Integers and Related Extremal Problems, McGraw-Hill, New York, 1970.

51. Samuelson, K., "International Information Transfer and Network Communication", in Cuadra, C. (ed.), Annual Review of Information Science and Technology, Interscience Publishers, vol. 6, 1971.

52. Serck-Hanssen, J., Optimal Patterns of Location, North Holland Publishing Co., Amsterdam, Holland, 1970.

53. Snyder, R., "A Note on the Location of Depots", Management Science, vol. 18, 1971, p. 97.

54. Spielberg, K, "Algorithms for the Simple Plant Location Problem with Some Side Conditions", Operations Research, vol. 17, 1969, p. 85.

55. Toregas, C., R. Swain, C. Revelle, and L. Bergman, "The Location of Emergency Service Facilities", Operations Research, vol. 19, 1971, pp. 1363-1373.

56. Vergin, R. C. and J. D. Rogers, "An Algorithm and Computational Procedure for Locating Economic Facilities", Management Science, vol. 13, 1966, p. 240.

57. Vollman, T. E. and E. S. Buffa, "The Facilities Layout Problem in Perspective", Management Science, vol. 12, 1966, p. 450.

58. Wagner, H., Principles of Operations Research, Prentice-Hall, New Jersey, 1969.

59. Wanty, J. R. M. "A Practical Application of the Warehousing Problem: Location of Depots in a Petroleum Company", Operational Research Quarterly, vol. 9, 1958, p. 149.

60. Wasserman, P. and E. Daniels, "Libray and Information Center Management", in Cuadra, C. (ed.), Annual Review of Information Science and Technology, Interscience Publishers, vol. 4, 1969.

61. Wheeler, J. L, "The Effective Location of Public Library Buildings", Occasional Paper No. 52, Graduate School of Library Science, University of Illinois, Urbana, Ill., 1958.

62. Wheeler, J. L. "A Reconsideration of the Strategic Location for Public Library Buildings", Occasional Paper No. 58, Graduate School of Library Science, University of Illinois, Urbana, Ill., 1967.

63. Wesolonsky, G. O. and R. F. Love, "The Optimal Location of New Facilities Using Rectangular Distances", Operations Research, vol. 19, 1971, pp. 124-130.

64. Young, H., "On the Optimum Location of Checking Stations", Operations Research, vol. 7, 1963, p. 721.

Ellwein, L. B. and P. Gray, "Solving Fixed Charge Location-Allocation Problems with Capacity and Configuration Constraints", AIIE Transactions, vol. 3, 1971, pp. 290-298.